ERDC/CRREL TR-08-24

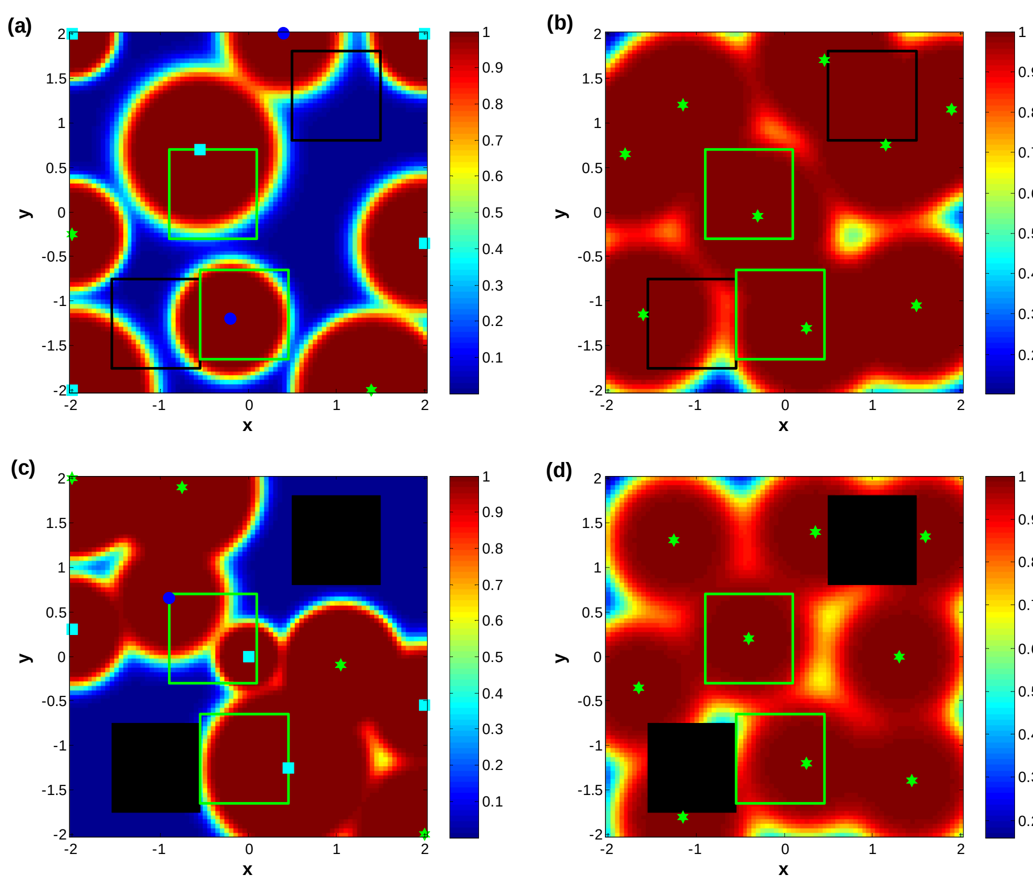
Cold Regions Research and Engineering Laboratory

US Army Corps of Engineers®
Engineer Research and Development Center

Optimal Sensor Placement with Terrain-Based Constraints and Signal Propagation Effects

Sergey N. Vecherin, D. Keith Wilson, and Chris L. Pettit

December 2008



COVER: Sensor locations in the presence of high-value objects (green squares) and obstacles (black squares).

ERDC/CRREL TR-08-24
December 2008

Optimal Sensor Placement with Terrain-Based Constraints and Signal Propagation Effects

Sergey N. Vecherin

Cold Regions Research and Engineering Laboratory
U.S. Army Engineer Research and Development Center
72 Lyme Road
Hanover, NH 03755-1290

Physics Department
New Mexico State University
Las Cruces, NM 88003

D. Keith Wilson

Cold Regions Research and Engineering Laboratory
U.S. Army Engineer Research and Development Center
72 Lyme Road
Hanover, NH 03755-1290

Chris L. Pettit

U.S. Naval Academy
Annapolis, MD 21402

Final report

Prepared for Headquarters, U.S. Army Corps of Engineers
Washington, DC 20314-1000

Under Work Unit B48G13

Abstract: The optimal sensor placement problem, as considered here, is to select the types and locations of sensors providing coverage at high-value terrain locations while minimizing a specified cost function. The cost function can reflect various disincentives, such as the actual cost of the sensors, the total number of sensors, and the probability that the sensor will be found and disabled by hostile actors. The probability of detection (at a certain probability of false alarm) is assumed to depend on terrain conditions and obstructions, and may be arbitrarily complex. Two strategies are described for finding the minimal number of sensors, and their locations that will satisfy given coverage preferences. The first is heuristic in nature and based on placing sensors one-by-one where the probability of detection is minimal. This strategy offers a rapid, but suboptimal solution. The second strategy is based on solution of the binary linear programming problem. For the case of fine spatial resolution that leads to large matrix dimensions, a fast algorithm for approximate solution of this problem is developed. The key features of this study are: 1) the probabilistic framework of sensor performance, 2) incorporation of the coverage preferences in the placement strategy, 3) realistic modeling and incorporation of the sensors' probability of detection, 4) multimodal sensor support, 5) a strict formulation of the optimal coverage problem, 6) development of a fast algorithm for approximate solution of the binary linear programming problem, and 7) introduction of a safe-mode concept.

Contents

Figures and Tables

Figures

Tables

Preface

This work was supported by the Engineer Research and Development Center (ERDC) Environmental Awareness for Sensor Employment (EASE) project. Work was performed under contract with the Oak Ridge Institute for Science and Education (ORISE).

This report was prepared under the general supervision of Dr. John M. Boteler, Chief, Signature Physics Branch, Cold Regions Research and Engineering Laboratory (CRREL); Dr. Justin B. Berman, Chief, Research and Engineering Division, CRREL; Dr. Lance D. Hansen, Deputy Director, CRREL; and Dr. Robert E. Davis, Director, CRREL.

The Commander and Executive Director of ERDC is COL Gary E. Johnston. The Director is Dr. James R. Houston.

1 Introduction

The problem of optimal sensor placement can be viewed as a generalization of the art gallery problem originally formulated as finding the minimal number and locations of guards so that each point of a gallery would be seen by at least one guard. This problem and its variations are well studied in computational geometry, and elegant solution algorithms have been developed (O'Rourke 1987; Shermer 1992). Because the problem is obviously identical to the problem of camera placement, it is not surprising that these algorithms are being used for security and surveillance. The problem can be generalized to include constraints on the camera's range, resolution, and field of vision (Erdem and Sclaroff 2004). Similar problems are encountered in many other areas, such as broadcasting or cellular tower placement (Eidenbenz 2002), robot motion planning (Elnagar and Lulu 2005; González-Banos et al. 1999), graphics (McKenna 1987), and computer vision and pattern recognition (Mittal and Davis 2008; Wixson 1994).

In this technical report, the original art gallery problem is generalized to include sensors of different modality (e.g., cameras and/or acoustic, infrared, seismic, and magnetic sensors). To describe and operate with multimodal sensors systematically, the probability of detection (at a certain probability of false alarm) is chosen as a sensor's performance measure (Wilson et al. 2008); it is universal[1], it directly relates to the receiver operating characteristic (ROC), which is a standard characteristic of a sensor, and it provides a natural way to formulate coverage preferences by directly assigning the desired probability of detection for each spatial point. (In this technical report, a standard terminology of signal detection is used: the probability of detection P_d is the probability that a sensor would detect a source when it is really there; the probability of false alarm P_{fa} is the probability that a sensor would detect a source when it is really not there; the probability of misdetection $P_{md} = 1 - P_d$ is the probability that a sensor would not detect a source when it is really there.) However, this brings the

[1] A probability of detection can be applied to any sensor. For example, an ideal camera has a probability of detection equal to one within its field of view and zero outside.

problem into probabilistic space, which generates two new aspects in comparison with the ordinary art gallery problem.

First, the ordinary art gallery problem and its variations require that any point within a specified area be seen by at least one guard (or camera). This requirement is unnecessary in the probabilistic framework of sensor performance. Indeed, to have sufficient coverage at a certain point (that is, a sufficiently high probability of detection, for example, over 0.95), it is not necessary to require that at least one sensor see it (that is, that at least one sensor have a probability of detection over 0.95 at this point). For example, two independent sensors can have a probability of detection at a certain point equal to 0.8 to see it with a probability of 0.96.

Second, the effects of signal propagation from the point where a signal was generated to the point where a sensor is located should be taken into account. In the ordinary art gallery problem, these effects are reduced to the geometric shadow zone: any point within the geometrical shadow of an occlusion for a certain camera is invisible for this camera. In probabilistic terms, this means that the probability of detection is zero for the shadow zone and is one for the field of view before the occlusion. This idealized picture is a consequence of the geometrical ray approximation for signal propagation. The probabilistic framework is capable of describing more complicated situations. The probability of detection depends on the signal and noise probability density functions (Burdic 1984). The signal and noise, while traveling toward a sensor, are subjected to all wave phenomena, such as scattering, attenuation (generally inhomogeneous), reflection from objects and boundaries, refraction, interference with other signals, and diffraction. Obstacles and terrain variations within a specified area of coverage may have different sizes, shapes, and attenuation factors (corresponding to totally obscured, semitransparent, and transparent areas). As a result, the probability of detection may be a highly complicated, anisotropic, inhomogeneous function of sensor and source locations.

The aim of this technical report is twofold. First, two multimodal sensor placement strategies that satisfy the specified coverage preferences and take into account the terrain-based and signal-propagation effects are presented. The coverage preferences are formulated by the direct assignment of the desired probability of detection to each point. For concreteness, the two-dimensional problem is considered in this report, although the developed approaches can be applied in the three-dimensional space. The first

strategy, called Strategy 1, is aimed at finding the minimal number of sensors; it is heuristic in nature and places sensors one-by-one in the worst covered spots (see section 3 for a strict definition of such spots). This strategy is similar to that described in Dhillon and Chakrabarty (2003) with three distinctions: 1) the probability of detection in the current study is modeled more realistically, taking into account the effects of signal propagation, rather than assuming the exponentially decaying law, 2) the coverage preferences are incorporated into the placement strategy rather than used as a stop criterion for a placement algorithm, and 3) two criteria are used simultaneously to decide what is the best place for the next placement. Strategy 1 yields a rapid but suboptimal solution (that is, it may not find the minimal number of sensors required). The second strategy, called Strategy 2, is based on a solution of the strictly posed binary linear programming optimization problem (Sierksma 2002). Such a formulation for the camera placement (a nonprobabilistic sensor framework) is presented in Erdem and Sclaroff (2004). This approach allows one to minimize total costs of the sensor network by specifying a cost function. The cost function reflects costs associated with the placement of a particular sensor to a particular place. It may be the actual cost of a sensor, the cost to mount and operate a sensor at a particular location, or vulnerability of a sensor. The cost function also could be chosen to minimize the total number of sensors (that is, the problem will be reduced to the same problem Strategy 1 aims to solve). Whatever cost function is chosen, Strategy 2 yields the global optimal solution, that is, there is no other configuration of sensors that would satisfy the coverage requirements with lesser costs. However, at high spatial resolutions, the optimization problem becomes very demanding of computational time (the problem belongs to the nondeterministic polynomial, more precisely, NP-complete, class of complexity that requires significant computational resources; Sierksma [2002]). In practice, a strict solution of this problem at large matrix dimensions is not obtainable. For example, in Erdem and Sclaroff (2004), the set of possible camera locations was quite sparse to allow one to solve the optimization problem. In this technical report, a fast algorithm for finding an approximate solution to this problem in the case of large dimensions is developed.

Second, a safe-mode concept for sensing is presented in this study. This concept takes into account that the probability of detection of a given sensor depends on the sensor's operational conditions, such as weather, terrain, and time of day. The same operational conditions may worsen the probability of detection of one sensor, but improve others. Defining safe

modes as having probability of detection of sensors within certain ranges, one can calculate the probability that at least one of the safe modes occurs under various operational conditions. A more detailed explanation of this concept is given in section 6. The concept is an adaptation of the voting logic inference, presented in Klein (2007), to the probabilistic sensor performance framework.

The report is organized as follows. Section 2 describes how the probability of detection can be modeled to take into account signal propagation effects. Strategy 1 is presented in section 3. Section 4 is devoted to Strategy 2 and the algorithm for an approximate solution of the binary linear programming problem. Section 5 presents the case study of the placement strategies for various coverage preferences and obstacles. The safe-mode concept is presented in section 6. Conclusions are drawn in section 7.

2 Probability of Detection with Signal Propagation Effects

To take into account signal propagation effects, the following physical model is adopted in this technical report. An object, subjected to detection and located at $\mathbf{r}_O$, generates multimodal signatures, called signals and denoted $s(\mathbf{r}_O, t)$, t being the time. These signals have random amplitude, shape, and duration. While traveling to a sensor location, the signals are subjected to scattering, attenuation, and other wave propagation effects; they may be corrupted by noise $n(\mathbf{r}_R, t)$, so that an imperfect signal $u(\mathbf{r}_R, \mathbf{r}_O, t) = s(\mathbf{r}_R, \mathbf{r}_O, t) + n(\mathbf{r}_R, t)$ is received by a sensor located at $\mathbf{r}_R$. The strict solution of what signal will be received at a sensor location would involve a solution of the wave equation with properly specified boundary and initial conditions, including conditions on obstacles and terrain inhomogeneities, which is beyond the scope of this report. Instead, some basic effects of signal propagation in the media with losses are taken into account, which, nevertheless, allow one to capture a realistic dependence of the probability of detection on distance. Note that this model is not aimed at enhancing or competing with a strict theory of signal propagation in random media (Tatarskii 1971; Ishimaru 1978; Ostashev 1997). In this theory, the goal is to find relationships between statistical properties of random media and statistical properties of the propagated signal. In the model presented in this technical report, the medium is considered to be nonrandom but inhomogeneous; that is, it may contain obstacles or other objects that are not random in nature, unlike turbulence. Taking turbulence into account could be a further enhancement of the proposed model.

The probability of detection $P_d(\mathbf{r}_R, \mathbf{r}_O)$ of a signal emitted at $\mathbf{r}_O$ by a sensor located at $\mathbf{r}_R$ depends on the energy of the signal and noise and can be calculated as follows (Burdic 1984):

$$P_d(\mathbf{r}_R, \mathbf{r}_O) = \int_{\gamma}^{\infty} p_{U(\mathbf{r}_R, \mathbf{r}_O)}(w)\, dw, \tag{1}$$

where p_U is the probability density function of energy $U(\mathbf{r}_R,\mathbf{r}_O)$ of the noisy signal $u(\mathbf{r}_R,\mathbf{r}_O,t)$, and γ is the detection threshold producing the prescribed probability of false alarm $P_{fa}(\mathbf{r}_R)$:

$$P_{fa}(\mathbf{r}_R) = \int_{\gamma}^{\infty} p_{N(\mathbf{r}_R)}(w)\,dw, \tag{2}$$

where p_N is the probability density function of energy $N(\mathbf{r}_R)$ of the noise $n(\mathbf{r}_R,t)$. Although Equation 2 allows one to specify P_{fa} as a function of $\mathbf{r}_R$, a usual choice would be to have a constant probability of false alarm for each sensor located at any $\mathbf{r}_R$ within an area being protected. To find γ, one may notice that it satisfies a requirement that the cumulative probability density function $F_{N(\mathbf{r}_R)}(\gamma) = \int_{-\infty}^{\gamma} p_{N(\mathbf{r}_R)}(w)dw$ would be equal to $1 - P_{fa}(\mathbf{r}_R)$. Then:

$$\gamma = F_{N(\mathbf{r}_R)}^{-1}\left(1 - P_{fa}(\mathbf{r}_R)\right), \tag{3}$$

where $F_{N(\mathbf{r}_R)}^{-1}$ stands for the inverse cumulative probability density function.

The signal $s(\mathbf{r}_R,\mathbf{r}_O,t)$ can be presented as $s(\mathbf{r}_R,\mathbf{r}_O,t) = s(\mathbf{r}_R,\mathbf{r}_O)s(t)$ because the wave equation allows separation of variables (in the absence of phase speed scatterers or, in practice, when they are negligibly weak; such an approximation is used in x-ray tomography, for example, where only the signal's attenuation is taken into account). For example, for a plane wave propagating in the two-dimensional isotropic homogeneous medium with losses (see, e.g., Ostashev [1997]):

$$s(\mathbf{r}_R,\mathbf{r}_O) = \exp\left(-\alpha(\omega)r + ik(\omega)r\right)/\sqrt{r} \tag{4}$$

and

$$s(t) = A\exp\left(-i\omega t\right), \tag{5}$$

where r is a distance between the transmission and reception $r = \left|\mathbf{r}_R - \mathbf{r}_O\right|$, $\alpha(\omega)$ is the amplitude attenuation factor, which, in general, may depend

on the frequency ω, i is the imaginary unit, $k(\omega)$ is the wave number, and A is the amplitude. This example assumes a harmonic signal with a single frequency ω. For more complicated signals, $s(t)$ is expressed in terms of the Fourier series or integral. Note, however, that for each of the harmonics in the Fourier spectrum, the spatial term $s(\mathbf{r}_R,\mathbf{r}_O)$ can be presented in the form given by Equation 4 with possibly distinct $\alpha(\omega)$ and $k(\omega)$. As a result, the signal's energy detected by a sensor, $S(\mathbf{r}_R,\mathbf{r}_O)$, which is proportional to $|s(\mathbf{r}_R,\mathbf{r}_O,t)|^2 = s(\mathbf{r}_R,\mathbf{r}_O,t)s^*(\mathbf{r}_R,\mathbf{r}_O,t)$, where * denotes complex conjecture, takes the form:

$$S(\mathbf{r}_R,\mathbf{r}_O) = A_1^2|s(\mathbf{r}_R,\mathbf{r}_O)|^2\int_0^T|s(t)|^2\,dt = \\ = \frac{\exp(-\beta r)}{r}S_0, \tag{6}$$

where

$$S_0 = A_1^2\int_0^T|s(t)|^2\,dt, \tag{7}$$

$\beta(\omega) = 2\alpha(\omega)$ is the energy attenuation factor, A_1^2 is a coefficient of proportionality, which may reflect the sensitivity of a sensor and the aperture of the signal reception, T is the duration of the signal, and S_0 is the energy contained in the temporal part of the signal $s(\mathbf{r}_R,\mathbf{r}_O,t)$. Because the duration and the amplitude of the signal $s(t)$ are random, S_0 is a random non-negative variable. Its probability distribution function is not easily obtainable. At a fixed T and normally distributed $s(t) \propto N(\mu_s(t),\sigma_s(t))$, its probability distribution function—after the integral is digitized and approximated by a sum with m terms—would be proportional to the non-central χ^2 distribution with m degrees of freedom and the parameter of noncentrality $\lambda = \sum_{l=1}^{m}(\mu_{sl}/\sigma_{sl})^2$. However, the authors are not aware of a defined probability distribution function for S_0 with random, arbitrary distributed m, and arbitrary distributed $s(t)$. In this technical report, it is assumed that S_0 can be well described by the normal distribution $N(\mu_{S_0},\sigma_{S_0})$ with positive $\mu_{S_0} > 3\sigma_{S_0}$. It follows from Equation 6 that $S(\mathbf{r}_R,\mathbf{r}_O)$ is also normally distributed with the mean μ_S and standard deviation σ_S given by the following formulas:

$$\mu_S(\mathbf{r}_R,\mathbf{r}_O)=\frac{\exp(-\beta r)}{r}\mu_{S_0},\ \sigma_S(\mathbf{r}_R,\mathbf{r}_O)=\frac{\exp(-\beta r)}{r}\sigma_{S_0}. \tag{8}$$

Note that for some sensors (e.g., seismic or magnetic) the power of the r in the denominator can be greater than one.

If energy attenuation is spatially nonuniform, that is $\beta=\beta(\omega,\mathbf{r})$, one can calculate integral attenuation $B(\mathbf{r}_R,\mathbf{r}_O)$ using a ray approximation and taking the integral along a ray's path:

$$B(\omega,\mathbf{r}_R,\mathbf{r}_O)=\int_{L_{\mathbf{r}_O\mathbf{r}_R}}\beta(\omega,\mathbf{r})dl, \tag{9}$$

where $L_{\mathbf{r}_O\mathbf{r}_R}$ is the length of a ray path from $\mathbf{r}_O$ to $\mathbf{r}_R$. Then, this $B(\omega,\mathbf{r}_R,\mathbf{r}_O)$ should be used instead of βr in Equation 6. Note that for straight paths and uniform β, Equation 9 yields the βr factor.

The noise energy $N(\mathbf{r}_R)$ is modeled taking into account the following considerations. The signal is corrupted by noise $n(\mathbf{r}_R,t)$ at the time and point of receiving. The energy of this noise accumulated in the signal of the duration T is:

$$N(\mathbf{r}_R)=A_2^2\int_0^T\left|n(\mathbf{r}_R,t)\right|^2dt, \tag{10}$$

where the constant A_2^2 is a coefficient of proportionality and may reflect the sensitivity of a sensor to noise. Following the same arguments as for the signal's energy, $N(\mathbf{r}_R)$ is assumed normally distributed with the mathematical expectation $\mu_N(\mathbf{r}_R)$ and standard deviation $\sigma_N(\mathbf{r}_R)$, such that $\mu_N>3\sigma_N$.

Finally, the energy of the noisy signal $u(\mathbf{r}_R,\mathbf{r}_O,t)=s(\mathbf{r}_R,\mathbf{r}_O,t)+n(\mathbf{r}_R,t)$ is:

$$U(\mathbf{r}_R,\mathbf{r}_O)=\int_0^T\left|u(\mathbf{r}_R,\mathbf{r}_O,t)\right|^2dt=S(\mathbf{r}_R,\mathbf{r}_O)+N(\mathbf{r}_R). \tag{11}$$

In Equation 11, the integrals over cross-terms s^*n and sn^* yield zero because the signal and the noise are assumed to be entirely incoherent. This

fact is well known, for example, in optics where the intensities of two incoherent sources are summed algebraically in contrast to coherent sources (Balanis 1989). Because S and N are independent normally distributed quantities, U is also normally distributed with the mean μ_U and variance σ_U^2 given by the following formulae:

$$\mu_U(\mathbf{r}_R,\mathbf{r}_O) = \mu_S(\mathbf{r}_R,\mathbf{r}_O) + \mu_N(\mathbf{r}_R), \;\; \sigma_U^2(\mathbf{r}_R,\mathbf{r}_O) = \sigma_S^2(\mathbf{r}_R,\mathbf{r}_O) + \sigma_N^2(\mathbf{r}_R). \tag{12}$$

At large distances from a source, when $r \to \infty$, the signal's mean and variance equal zero (see Equation 8), so that pure noise is detected by a sensor. In this case, $\mu_U = \mu_N$ and $\sigma_U = \sigma_N$, as follows from Equation 12, the probability density functions p_U and p_N coincide, and $P_d = P_{fa}$ (all detections are false alarms). Figure 1 shows positions of three probability density functions, for the signal, noise, and noisy signal, when a sensor is not infinitely distant from a source.

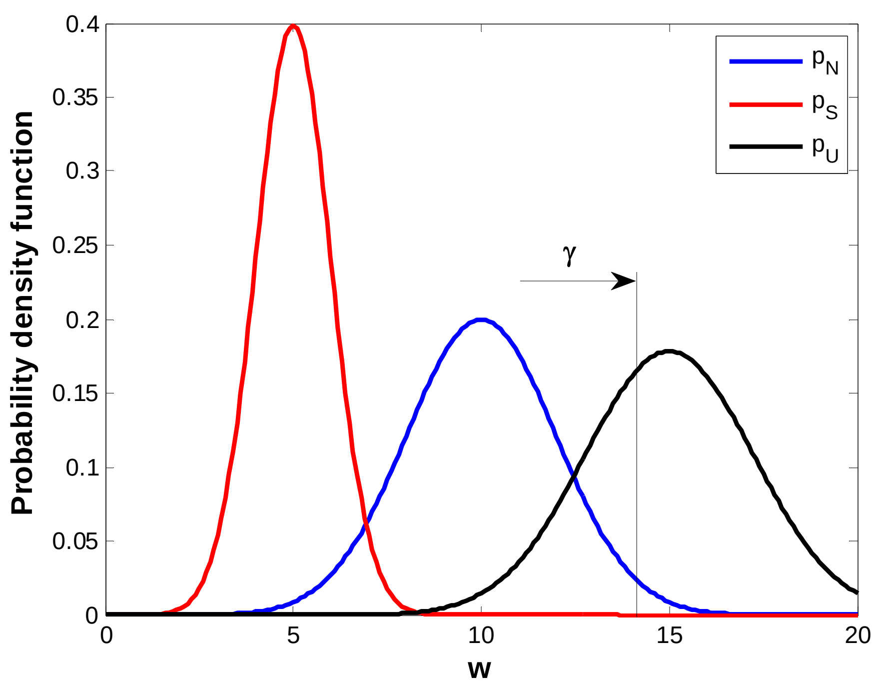


Figure 1. Probability density functions of the signal energy p_S (left), noise energy p_N (middle), and noisy signal energy p_U (right). The horizontal axis represents dimensionless energy w. The source is quite distant from a receiver so that its energy is less than the noise energy. The γ is found as a quantile of p_N corresponding to a prescribed probability of false alarm. The probability of detection is found by the integration of p_U from γ to ∞.

For finite r, the amplitude and variance of a signal are not zeros (see Equation 8), $\mu_U > \mu_N$ and $\sigma_U > \sigma_N$, which causes the p_U curve to move to the right relative to the noise distribution function, and P_d becomes greater than P_{fa}. In Figure 1, w (the horizontal axis) denotes dimensionless energy and the vertical line shows the value of γ. At some short distances to a source, the p_U curve lies entirely in the right-hand side area from p_N and $P_d = 1$.

A one-dimensional spatial section of the probability of detection of a signal emitted at x, i.e., $\mathbf{r}_O = (x,0)$, by a sensor located at $\mathbf{r}_R = (0,0)$ is shown in Figure 2.

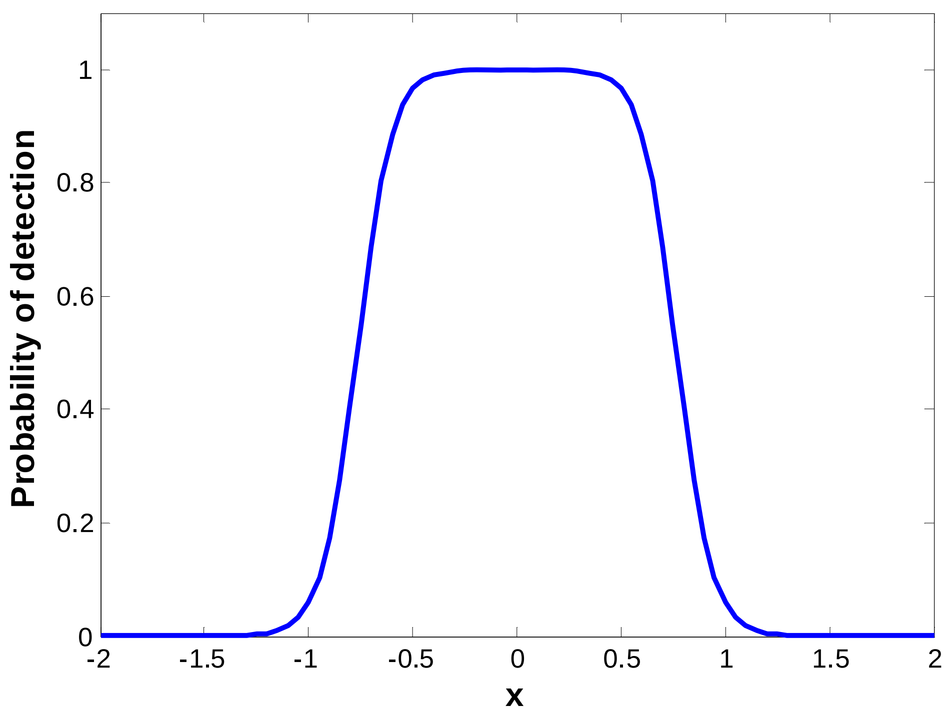


Figure 2. One-dimensional spatial section of the probability of detection of a signal emitted at $\mathbf{r}_O = (x,0)$ by a sensor located in the origin.

For each $x \subset [-2,2]$, the mean and variance of the signal's energy were calculated in accordance with Equation 8 with $r = |\mathbf{r}_R - \mathbf{r}_O| = |x|$ and constant β. These mean and variance define the location and shape of the red curve (the signal) in Figure 1, while the blue curve (the noise) does not move (the

noise's mean and variance are independent of the distance between a source and a receiver by assumption). The resulting black curve (the noisy signal) also moved and changed its shape because its mean and variance changed in accordance with Equation 12. The γ was determined from the blue curve (Equation 3) at probability of false alarm $P_{fa} = 10^{-6}$. The probability of detection (the vertical axis) was obtained by the integration of the black curve from this γ to infinity (Equation 1). As one can see, the spatial dependence of the probability of detection falls quite suddenly beyond a certain area of visibility (the detection with a high degree of probability). This dependence is not likely due to the exponential or Gaussian function. A numerical experiment with the noncentral χ^2 probability density function for $U(\mathbf{r}_R, \mathbf{r}_O)$ and $N(\mathbf{r}_R)$ (at a fixed signal duration T) has been carried out as well. The probability of detection obtained was similar to that shown in Figure 2 with minor quantitative distinctions while the numerical instability[2] and computational time were significantly greater. Thus, the Gaussian probability distributions do not change principal results and, therefore, will be used in this technical report.

[2] Calculations of the inverse noncentral chi-squared distribution functions involve a summation of infinite series, which have poor convergence and precision at certain combinations of their arguments (depending on the chosen probability of false alarm and signal and noise magnitudes and variances).

3 Strategy 1: "Cover the Worst First"

The strategy described in this section is similar to that reported in Dhillon and Chakrabarty (2003). It belongs to the class of "greedy" algorithms, meaning that only one sensor at a time is being optimized. Although such algorithms do not guarantee that the final sensor's network will be optimal, they are widely used for their speed. Strategy 1 is heuristic in nature and based on the idea that sensors are placed one-by-one, each in the place where it is most needed. In Dhillon and Chakrabarty (2003), this place is determined as the place with the minimal probability of detection provided by a current sensor network. The probability of detection is assumed to be described by the exponential function $\exp(-ar)$. The coverage preferences are formulated as the desired probability of detection at each spatial point. This allows one to assign a higher desired probability of detection to areas of high importance and a lower one to areas of low importance. If there are no specific preferences, one can assign a constant desired probability of detection to all points. If there is a certain area with higher coverage preferences, then one continues to add sensors until the preferences are satisfied. In other words, the preferences serve as a stop criterion and are not incorporated in the placement strategy.

Strategy 1 described in this section has three enhancements. First, the preferences are incorporated into the strategy. This is accomplished by assuming that the worst covered spot is that where the discrepancy between the current and desired coverage is the largest. Second, a more realistic function of spatial dependence of the sensor's probability of detection is used. This function (see Figure 2) falls quite suddenly beyond a certain area and, therefore, there are many spatial points that are covered equally poorly. Therefore, it is worthwhile to have another criterion to decide which point among those is the best in which to place a sensor. A geometrical criterion is used in our approach, which is the third enhancement. It is assumed that, among those points, the most distant point from all sensors is the best. If there are many best points, one can take any, for example, the first one. Taking these considerations into account, Strategy 1 can be formulated as follows.

Step 1. Calculate the current probability that at least one sensor in a network detects a source, $P_c(\mathbf{r}_O;\mathbf{r}_{R_1},\ldots,\mathbf{r}_{R_M})$, where M is the current number of sensors, $(\mathbf{r}_{R_1},\ldots,\mathbf{r}_{R_M})$ are their locations, and $\mathbf{r}_O=(x,y)$ indicates the possible locations of a source (that is, each spatial point within a protected area). P_c is considered as a function of $\mathbf{r}_O$ while $\mathbf{r}_{R_1},\ldots,\mathbf{r}_{R_M}$ are fixed parameters. To simplify notations, these parameters will be omitted whenever this will not confuse the derivations. It is assumed that sensors detect a source independently. This assumption allows one to calculate $P_c(\mathbf{r}_O;\mathbf{r}_{R_1},\ldots,\mathbf{r}_{R_M})$ very efficiently through the probability of misdetection $P_{md}=1-P_c$ (not to be confused with the probability of false alarm P_{fa}). $P_{md}(\mathbf{r}_O;\mathbf{r}_{R_1},\ldots,\mathbf{r}_{R_M})$ is the probability that all M sensors fail to detect a source simultaneously. Because sensors are assumed to be independent, P_{md} can be expressed in terms of the probability of misdetection of a single sensor $P_{md_i}(\mathbf{r}_O;\mathbf{r}_{R_i})$, $i=1,\ldots,M$:

$$P_{md}(\mathbf{r}_O;\mathbf{r}_{R_1},\ldots,\mathbf{r}_{R_M})=\prod_{i=1}^{M}P_{md_i}(\mathbf{r}_O;\mathbf{r}_{R_i}), \tag{13}$$

where the probability of misdetection of a single sensor is calculated using the single sensor's probability of detection $P_d(\mathbf{r}_O;\mathbf{r}_{R_i})$ given by Equation 1:

$$P_{md_i}(\mathbf{r}_O;\mathbf{r}_{R_i})=1-P_d(\mathbf{r}_O;\mathbf{r}_{R_i}). \tag{14}$$

Then, the probability that at least one sensor of M would detect a source can be found easily:

$$P_c=1-P_{md}. \tag{15}$$

If no sensors are placed yet ($M=0$), one sets $P_c=0$ for each point.

Step 2. Calculate the discrepancy ε between the current and desired probabilities of detection:

$$\varepsilon(\mathbf{r}_O)=P_c(\mathbf{r}_O)-P_{pr}(\mathbf{r}_O), \tag{16}$$

where $P_{pr}(\mathbf{r}_O)$ is the desired probability of detection of a source at $\mathbf{r}_O$. In principal, this criterion alone already allows one to find the best location for another sensor $\mathbf{r}_{R_{M+1}}$, which would correspond to the minimum of ε:

where $P_{pr}(\mathbf{r}_O)$ is the desired probability of detection of a source at $\mathbf{r}_O$. In principal, this criterion alone already allows one to find the best location for another sensor $\mathbf{r}_{R_{M+1}}$, which would correspond to the minimum of ε:

$$\mathbf{r}_{R_{M+1}} = \arg\min[\varepsilon(\mathbf{r}_O)]. \tag{17}$$

If there are many points where ε reaches its minimum, one takes the point in the middle of that set. This approach is used in the numerical experiment, described in this report, in the absence of other sensors (that is, to find the best location for the first sensor). If there are other sensors ($M > 0$), criterion ε can be enhanced by a geometrical criterion, as shown in Step 3.

Step 3. Calculate criterion d_0. Criterion d_0 takes into account current locations of sensors and defines the best point for another sensor as the most remote point from all existing sensors. More precisely, for each spatial point $\mathbf{r}_O$ of possible source locations, one finds the distance to the nearest sensor $d_0(\mathbf{r}_O) = \min\{d(\mathbf{r}_O, \mathbf{r}_{R_1}), \ldots, d(\mathbf{r}_O, \mathbf{r}_{R_M})\}$, where $d(\mathbf{r}_O, \mathbf{r}_{R_i})$ are distances from $\mathbf{r}_O$ to sensors located at $\mathbf{r}_{R_i}$. Then, according to criterion d_0, the best point is that where d_0 reaches its maximum:

$$\mathbf{r}_{R_{M+1}} = \arg\max[d(\mathbf{r}_O)]. \tag{18}$$

Criterion d_0 alone, as well as criterion ε, can be used to place sensors. In this case, the locations can be found in advance purely from geometrical considerations, without any characteristics of sensors required.

Step 4. Calculate a supercriterion C that combines both the ε and d_0 criteria together. To do this, one needs to normalize ε and d_0 so that they have the same range of variability. The following normalization yields the range [0,1] for normalized ε and d_0 (C_1 and C_2, correspondingly):

$$\begin{aligned} C_1(\mathbf{r}_O) &= \frac{\max[\varepsilon(\mathbf{r}_O)] - \varepsilon(\mathbf{r}_O)}{\max[\varepsilon(\mathbf{r}_O)] - \min[\varepsilon(\mathbf{r}_O)]}, \\ C_2(\mathbf{r}_O) &= \frac{d_0(\mathbf{r}_O)}{\max[d_0(\mathbf{r}_O)]}, \end{aligned} \tag{19}$$

Supercriterion C is nothing but the sum of C_2 and C_1. Thus:

$$\mathbf{r}_{R_{M+1}} = \arg\max\left[C_2(\mathbf{r}_O) + C_1(\mathbf{r}_O)\right]. \tag{20}$$

Supercriterion C operates as follows. If, for example, there are two points with the same discrepancy between the current and desired probability of detection, then the most distant point from all sensors will be selected. And vice versa, if there are two points equally remote from all other sensors, the one with the minimal ε (or, equivalently, with the maximal C_1, which corresponds to the higher discrepancy between the current and desired probabilities) will be selected. If there are many points with the same value of C, any could be selected, for example, the first one. To vary the importance of C_2 or C_1 in the final decision, one can assign different weight factors W_2 and W_1 to criteria C_2 and C_1.

Step 5. Once the new placement point $\mathbf{r}_{R_{M+1}}$ is known, the last step is to determine what type of sensors should be placed there. Because the goal is to find the minimal number of sensors, then a sensor with a wider area of coverage (if placed at $\mathbf{r}_{R_{M+1}}$) will be the choice. Now the current number of sensors is $M+1$, and one repeats the algorithm from Step 1 until all points are covered with the desired probability of detection (or greater) or the number of available sensors is reached.

The described strategy has several advantages. First, it is very fast. On a spatial grid 81×81 points, it takes a fraction of a second to place a virtually unlimited number of sensors (on Intel Xeon 2.66 GHz PC with 4 Gb of RAM). Second, to place a new sensor, one does not need to remount and replace already existing sensors. In other words, the strategy allows an expansion of an existing network. If, for example, a current area being covered becomes larger, one can take an existing sensor network as a starting point and expand it to match the new area. And third, the strategy can be limited by a maximal number of available sensors even if the desired probability of detection was not reached. As one will see below, the strict solution does not allow such a limitation.

The disadvantage of this strategy is that it is suboptimal. Although the solution found is much better than a random placement, as demonstrated in Dhillon and Chakrabarty (2003) for the two-dimensional case, it may not have the widest possible area of coverage for a given number of sensors;

or, if there is no limitation on the total number of sensors, the solution may not yield the minimal possible number of sensors to satisfy the coverage preferences. To illustrate this point, imagine that one needs to cover uniformly a one-dimensional segment by a minimal number of sensors. Suppose the sensors are identical and have the radius of visibility equal to one-third of the segment's length. According to Strategy 1, the first sensor will be placed in the center of this segment, covering two-thirds of its length, the second sensor will be placed in one of the segment's vertices (because they are most remote from the center and have the worst coverage), and the third one will be placed in the other vertex. Thus, the answer would be that three sensors are required. Obviously, the strict optimization would place only two sensors (for example, in the positions that divide the segment into three equal parts).

4 Strategy 2: "Do the Job with the Minimal Cost"

Strategy 2 was developed to overcome the nonoptimality of Strategy 1. It is based on a solution of the strictly posed optimization problem, and guarantees determination of a global minimum (if it exists) of costs specified by a cost function. The cost function may reflect the actual costs of sensors, or other disincentives, such as the total number of sensors, the sensors' vulnerability, or their reliability. For fine spatial resolution, which leads to large matrix dimensions, however, the strict solution of this problem is not practically obtainable. The same strict problem formulation for camera placement can be found, for example, in Erdem and Sclaroff (2004). In this work, the authors were forced to have a rather coarse spatial grid for possible camera locations to be able to solve the problem. In this technical report, an approximate solution of the strictly posed optimization problem is found for large dimensions, allowing fine resolution for both the sensor placement and the area to be covered.

Strict formulation of the optimal coverage problem

In this subsection, it is shown how the problem of optimal coverage in the probabilistic sensor performance framework can be strictly formulated as the binary linear programming problem. Let us introduce a column-vector $\mathbf{p}$. The length of vector $\mathbf{p}$ equals the number of possible sensor locations K; only a few of them will be chosen as optimal ones. Furthermore, let us require that the elements of this vector could take either 0 or 1 values only. Then, vector $\mathbf{p}$ can be treated as the indicative vector: its 0 element in the k th position shows that no sensors should be placed in the k th possible point of location while 1 in this position indicates that a sensor should be placed there. If the goal is to find a minimal number of sensors, then the optimal vector $\mathbf{p}_0$ would have a minimal number of ones. Their positions in this vector indicate where to place sensors. Mathematically, this requirement is formulated as finding a minimum of a scalar linear function $F = \mathbf{f}^T\mathbf{p}$, where $\mathbf{f}$ is a column vector of costs associated with a particular sensor and T indicates the transposition. For a minimal number of sensors, $\mathbf{f} = [1;1;\ldots;1]$, where the semicolon between elements denotes that these elements are arranged in a column. But $\mathbf{f}$ can consist of

other values, for example, f_k can be equal to the actual costs of placing a sensor in the k th possible point or it could be the probability that a sensor will be found and disabled at this point. From this perspective, the F function, which is subject to minimization, is nothing but the total cost of placing the sensors. If there are several types of sensors, then there is a vector of possible locations for each of them, and the length of the $\mathbf{p}$ vector changes accordingly. For example, if the possible locations for two types of sensors are the same (e.g., length K), then the $\mathbf{p}$ vector would be length $2K$, and the number and positions of ones would indicate how many and what type of sensors are required. The cost function $\mathbf{f}$, in this case, also is length $2K$ and reflects the costs for the first and second types of sensors.

To impose the constraints of coverage, let us note that requiring the probability of detection P_d greater or equal to the given preferences P_{pr} is equivalent to requiring the probability of misdetection P_{md} less or equal to the allowed probability of misdetection: $P_{md} \le 1 - P_{pr}$. According to Equation 13, P_{md} is a product of the individual P_{md_i}'s. Therefore, taking the logarithm from both sides of this inequality, one has for each point $\mathbf{r}_O$:

$$\sum_{i=1}^{M} \ln P_{md_i}(\mathbf{r}_O, \mathbf{r}_{R_i}) \le \ln\left(1 - P_{pr}(\mathbf{r}_O)\right), \tag{21}$$

where $P_{md_i}(\mathbf{r}_O, \mathbf{r}_{R_i})$ are given by Equation 14. In this context, M equals the number of ones in the vector $\mathbf{p}$. If the total number of possible source locations $\mathbf{r}_O$ is Q, the last equation can be rewritten, in matrix notations,

$$\mathbf{Dp} \le \mathbf{b}, \tag{22}$$

where $\mathbf{b} = \ln\left(1 - P_{pr}(\mathbf{r}_O)\right)$, the k th column of matrix $\mathbf{D}$ (size $Q \times K$) represents the logarithm of the probability of misdetection for a sensor placed in the k th point, $\ln P_{md_k}(\mathbf{r}_O, \mathbf{r}_{R_k})$, and the inequality reads as an element-by-element comparison. Note that the number of terms actually selected for the summation in Equation 22 is defined by the number and position of ones in the vector $\mathbf{p}$.

Thus, the problem of the optimal coverage can be formulated as the minimization of total costs with constraints that reflect the coverage prefer-

ences and restrict $\mathbf{p}$ to be a binary vector. That is, one seeks an optimal vector $\mathbf{p}_0$ such that:

$$\mathbf{p}_0 = \arg\min \mathbf{f}^T\mathbf{p}, \tag{23}$$

and

$$\mathbf{Dp} \le \mathbf{b},\ \ p_k = \{0,1\},\ \ k = 1,\ldots,K. \tag{24}$$

This is one of the forms of the binary linear programming problem. If the problem is feasible (that is, the constraints permit at least one solution) then its solution is guaranteed to be the global minimum of the total costs (because F is a linear function of p; Sierksma [2002]). This means there is no other configuration of sensors satisfying the coverage preferences that would decrease the total costs further. In the example with a one-dimensional segment, introduced in section 3, a strict solution of this problem would place only two sensors within the segment.

However, for large dimensions of vector $\mathbf{p}$ and matrix $\mathbf{D}$, a strict solution is difficult to obtain. The binary linear programming problem does not have a quick solution, unlike its nonbinary counterpart, which can be solved effectively by simplex or interior point methods. To give insight why the binary requirement complicates the solution, let us consider a basic idea of the boundary and branch method, one of the most widely used methods for the binary optimization problems. The details can be found, e.g., in Sierksma (2002). At first, a nonbinary problem is solved with the constraints that all elements of $\mathbf{p}$ must be within the interval $[0,1]$. Then, each noninteger element of this solution is replaced by 0, which generates one branch of the possible binary solution, and 1, which generates another branch of the possible binary solution. Therefore, if the number of noninteger elements is L, the total number of branches to investigate is 2^L. In this method, a smart way to investigate and cut unnecessary branches is used, based on the constraints (matrix $\mathbf{D}$). In the numerical simulation described in this report, the area in consideration was partitioned into 81×81 spatial cells, so that the number of possible sensor locations and possible signal emissions was $81^2 = 6561$. Hence, the size of $\mathbf{D}$ was [6561 x 6561] and the length of $\mathbf{p}$ was 6561. In the worst case, all elements of $\mathbf{p}$, after solving the nonbinary problem, can be noninteger, and one will need to investigate 2^{6561} branches. Regardless of how smart the algorithm of branch selection is, the total numbers of branches and constraints are

large enough to avoid finding any solution with reasonable computational effort. An algorithm proposed in the next subsection finds an approximate (almost optimal) solution to this problem in less than one minute using an Intel Xeon 2.66 GHz PC with 4 Gb of RAM.

Approximate solution for minimal number of sensors

In this subsection, an algorithm for approximate solution to the binary linear programming problem is presented for the case when $\mathbf{f} = [1;1;\ldots;1]$; that is, when the goal is to minimize the total number of sensors. A generalization of this algorithm for other $\mathbf{f}$'s will be given in the next subsection. The approximate solution, presented in this subsection, converts Strategy 2 into a "greedy" algorithm (only one sensor at a time is optimized). However, Strategy 2 places sensors in a significantly different way than Strategy 1 as will be demonstrated in Section 5.

One can note that, for the binary vector $\mathbf{p}$, the minimal and maximal values of total costs $F = \mathbf{f}^T\mathbf{p}$ are known in advance. These are 0 (when $\mathbf{p} = \mathbf{0}$) and K (when $\mathbf{p} = \mathbf{1}$). The only reason why $\mathbf{p} = \mathbf{0}$ cannot be declared as the solution is the constraint $\mathbf{Dp} \le \mathbf{b}$, which is not satisfied by this $\mathbf{p}$ (if the constraint is satisfied, then the optimal solution is $\mathbf{p}_0 = \mathbf{0}$). Note that if this constraint is not satisfied by $\mathbf{p} = \mathbf{1}$ either, then the problem is infeasible, i.e., there is no solution to this problem in the binary space for $\mathbf{p}$.[3] Thus, one can verify very quickly whether the problem is feasible.

If the problem is feasible, one needs to distribute as few ones as possible among positions in the $\mathbf{p}$ vector (which is filled in by zeros otherwise). This would yield a strict solution. The idea of the algorithm for approximate solution is to do it consecutively to avoid investigating all possible combinations. The placement of a single 1 in any position in $\mathbf{p}$ is equivalently poor from the standpoint of the total cost F because it will equally increase its value (from 0 to 1). However, the constraints will be satisfied differently. Let $\hat{\mathbf{p}}$ be a trial solution that equals $\mathbf{p}$ with the extra 1 added. The best position of 1 is that where the sum over the positive elements of $\mathbf{\Delta} = \mathbf{D}\hat{\mathbf{p}} - \mathbf{b}$ is minimal:

---

[3]This means the coverage preferences are not satisfied even if one places sensors at all possible points. Such a situation is likely to happen when $K << Q$, i.e., when a large area is supposed to be covered with a few allowed sensor locations.

$$\sum_{i=1}^{I} \Delta_i^+(\hat{\mathbf{p}}) \mapsto \min,$$
$$\Delta^+(\hat{\mathbf{p}}) = \{\Delta_q(\hat{\mathbf{p}}):\ \Delta_q(\hat{\mathbf{p}}) > 0\},\ q = 1,\ldots,Q, \tag{25}$$

where I is the number of points where the coverage is not satisfied. If at some position of 1 there are no such points ($I = 0$), then all coverage preferences are satisfied, and the optimal vector $\mathbf{p}_0$ equals the trial vector $\hat{\mathbf{p}}$ where this happens. Thus, the algorithm for an approximate solution can be formulated as follows.

Step 1. Verify that the problem is feasible. If it is not, then there is no solution. If it is, set $\mathbf{p}_0 = \mathbf{0}$.

Step 2. Verify that there is at least one point where the coverage is not satisfied. If the coverage preferences are satisfied everywhere, return $\mathbf{p}_0$. If not, proceed as follows. Set a trial vector $\hat{\mathbf{p}} = \mathbf{p}_0$. Let the possible number of sensor locations be K. (These possible sensor locations correspond to zeros in $\hat{\mathbf{p}}$). Then, consecutively set zero elements in $\hat{\mathbf{p}}$ to one and calculate $\mathbf{\Delta} = \mathbf{D}\hat{\mathbf{p}} - \mathbf{b}$. Find the positive elements of $\mathbf{\Delta}$ and sum them over. Memorize this sum in $E(k)$, $k = 1,\ldots,K$. If at some k there will not be positive elements of the residual ($I = 0$), set $p_{0k} = 1$ and return $\mathbf{p}_0$. The solution is found. If not, go to Step 3.

Step 3. Find the best k_0 where $E(k)$ reaches its minimum: $k_0 = \arg\min E(k)$. Set $p_{0k_0} = 1$.

Step 4. Exclude unnecessary points from possible sensor locations. Obviously, at least point k_0 should be excluded (one does not need to place two sensors at the same point). But it is not the only possibility. One can find all spatial points where the coverage requirements are already satisfied (by placement of one sensor at a certain point corresponding to k_0). Namely, these points are defined by the row indices of matrix $\mathbf{D}$ at which $\mathbf{D}\mathbf{p}_0 \le \mathbf{b}$. If some spatial points of possible sensor locations also belong to the area where the coverage is already satisfactory, then all these points should be excluded from further consideration because it does not make much sense to place another sensor in this area. This significantly reduces the number of possible sensor locations K (note that the spatial point corresponding to k_0 will be excluded automatically). Go to Step 2.

In the worst case, the algorithm will converge to the most nonoptimal solution with $\mathbf{p}_0 = \mathbf{1}$.

This strategy can be viewed as walking from the ultimate optimal point $\mathbf{p} = \mathbf{0}$ (where the cost F is minimal but constraints are not satisfied) to the most nonoptimal point $\mathbf{p} = \mathbf{1}$ (where the cost F is maximal but the constraints are guaranteed to be satisfied) along the ribs of a K-dimensional hypercube, hoping to satisfy the coverage requirements before the most nonoptimal point is reached, as illustrated in Figure 3.

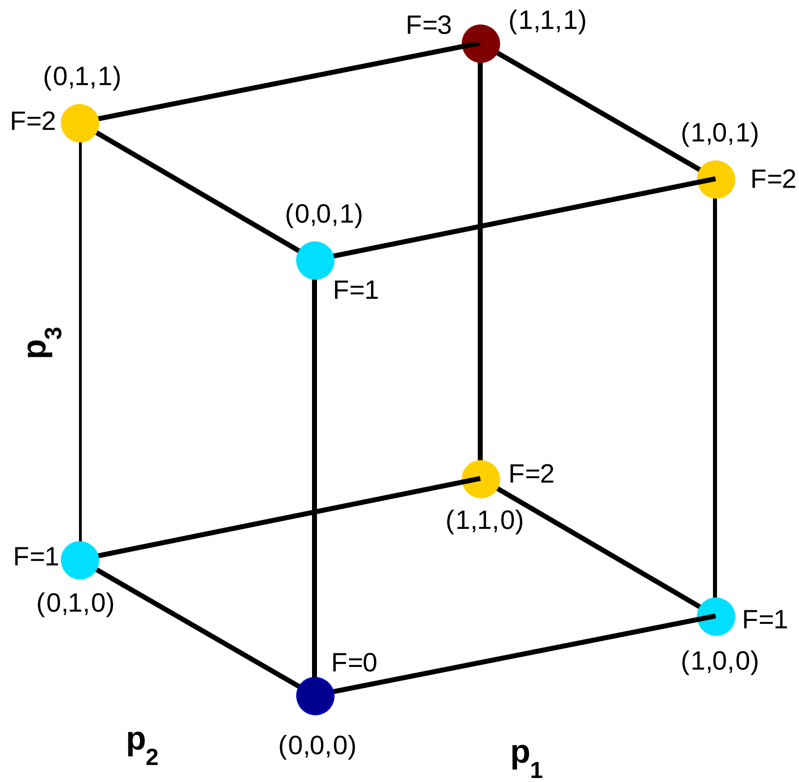


Figure 3. Strategy 2 guides a walker from the most optimal node (0,0,0), where the constraints are not satisfied, to the most nonoptimal node (1,1,1), where the constraints are satisfied, without jumping to nonadjacent nodes. The colors of nodes reflect the value of total cost *F*.

In this example, $K = 3$, so that $\mathbf{p} = (p_1, p_2, p_3)$. The color of the nodes corresponds to the value of the total cost F. The lowest cost ($F = 0$) corresponds to the blue node at $(0,0,0)$. The highest cost ($F = 3$) is at the red node at $(1,1,1)$. One needs to walk from $(0,0,0)$ to $(1,1,1)$ without jumping (this is the requirement of the consecutive placement; if jumping to nonadjacent nodes were allowed, a strict solution would result). Because all

neighbors around the blue node equally increase the cost F, all of them are of the same color. Among these equally poor directions, the algorithm selects the one that brings a walker closer to satisfaction of the problem constraints.

This simple strategy provides surprisingly good results, as demonstrated in section 5. Among many possible sensor locations, it quickly finds a small number satisfying the coverage preferences. But the solution may be suboptimal, as a consequence of the consecutive placement (i.e., sensors are placed one-by-one). In the one-dimensional example, introduced in section 3, the first sensor would be placed in the center of a segment, the second one in the uncovered segment of one of the halves, and the third one in the uncovered segment of the other half. At first glance, this is very similar to the results of Strategy 1, but the distinction will be clear in the two-dimensional case, as demonstrated in section 5. The advantage of the consecutive placement is that the total number of sensors available can be incorporated into the stop criterion of the algorithm.

Generalization to entirely positive or negative cost functions

Strategy 2 can be generalized for arbitrary cost functions with all positive or negative elements. This is a very practical situation because the costs of different sensors (whatever they reflect) are usually of the same sign. If all elements of $\mathbf{f}$ are negative, all inference of Strategy 2 remains the same with the only distinction that $\mathbf{p} = \mathbf{1}$ is the ultimate optimal solution, $\mathbf{p} = \mathbf{0}$ is the most nonoptimal one, and one fills the ultimate optimal solution with zeros rather than with ones. In terms of Figure 2, a walker now goes in the opposite direction from $(1,1,1)$ to $(0,0,0)$. For concreteness, the positive cost functions will be considered in this subsection: $\mathbf{f} \geq \mathbf{0}$.

In this case, as before, $\mathbf{p} = \mathbf{0}$ and $\mathbf{p} = \mathbf{1}$ are the ultimate optimal and nonoptimal solutions, correspondingly. However, different trial solutions in Step 2 are not equivalent now from the total cost point of view (all neighbor nodes would have different colors in Figure 3). Therefore, one should trade off between growing costs and coverage satisfaction. Step 2 should change as follows. In addition to $E(k)$, which reflects the quality of coverage, one needs to calculate and memorize the total costs $F(k) = \mathbf{f}^T \hat{\mathbf{p}}$ corresponding to the trial solution $\hat{\mathbf{p}}$. The most appealing point for a sensor placement would be that where the coverage is maximal while the cost

increase is minimal. To combine two criteria together, a supercriterion G can be introduced, similarly to the supercriterion C in Strategy 1:

$$G(k) = \frac{E(k) - \min[E(k)]}{\max[E(k)] - \min[E(k)]} + \frac{F(k) - \min[F(k)]}{\max[F(k)] - \min[F(k)]}, \quad (26)$$

where normalization is needed to make these two competitive factors equally important. To make them unequally important, one can introduce weight factors for each of the terms in Equation 26. The best position is defined now as $k_0 = \arg\min G(k)$.

5 Numerical Experiment

The goal of the numerical experiment was to compare Strategy 1 and Strategy 2 under various coverage preferences and obstacle locations. The area being protected (a 2-x-2 square in conditional units) was partitioned into 81-x-81 grid cells. For Strategy 1, there were three types of sensors, each with a specific attenuation constant β that defines the radius of the sensor's coverage (see Equation 8). Moreover, it was assumed that these β's depend on a placement point. For example, a type 1 sensor placed at one point could be better than a type 2 sensor at the same point, although the situation may reverse for another point. The values of these β's were randomly chosen from the interval [0,1]. A sensor of type 1 had a denominator proportional to r^2 rather than r, in contrast to other types. The energies of signal and noise were modeled in accordance with section 2. For Strategy 2, only one type of sensors was available with random, point-independent β. This allowed one to better illustrate the main features of Strategy 2 (the features of Strategy 1 can be seen without this restriction). As mentioned in section 4, Strategy 2 allows a strict expansion to multimodal sensors. The goal was to minimize the total number of sensors, which was restricted to nine, again for illustrative purposes. Without this restriction, both strategies would eventually satisfy the coverage preferences (with a different number of sensors). The probability of false alarm P_{fa} was equal to 10^{-6} for each sensor.

Figure 4, (a) and (b), shows the locations obtained with the use of Strategy 1 and Strategy 2, correspondingly, for the case of empty space. The color represents the probability that at least one sensor will detect a source. Different markers in Figure 4 (a) correspond to different types of sensors selected by Strategy 1. The coverage preferences were 0.95 for each spatial point. As one can see, neither of the strategies satisfied the coverage preferences by the means of nine sensors. However, the area covered by Strategy 2 is remarkably larger than that covered by Strategy 1. This can be interpreted to mean that Strategy 2 yields more optimal results than Strategy 1. This tendency holds in more complicated cases, as seen below.

Figure 5 presents the sensor locations and coverage with two high-value areas indicated as green squares. The coverage preferences were 0.95 inside these areas and 0.8 outside. Again, Strategy 2 places sensors more

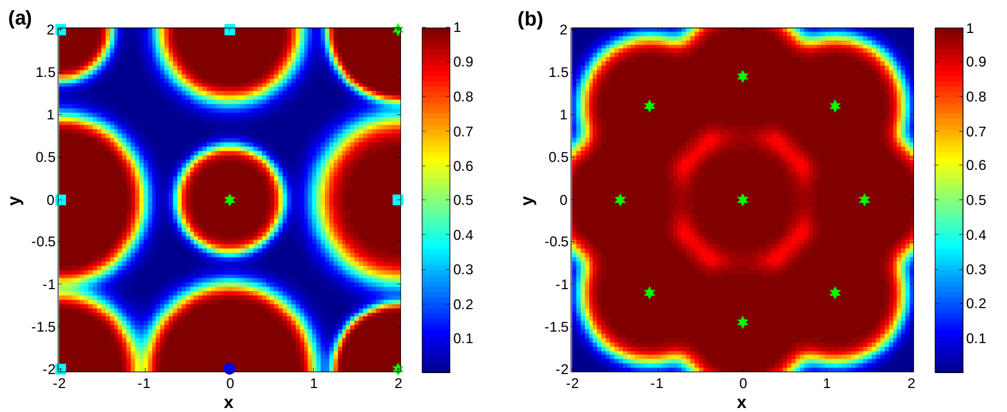


Figure 4. Probability of detection by at least one of the sensors. Sensor locations obtained by (a) Strategy 1 and (b) Strategy 2. The coverage preferences were 0.95 for each point, and the total number of available sensors was limited to nine. Different markers (a) correspond to different sensor types.

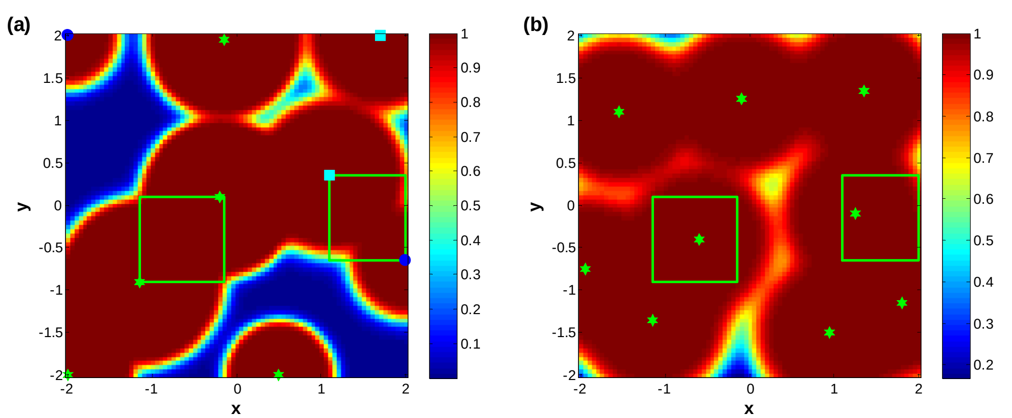


Figure 5. Coverage provided by (a) Strategy 1 and (b) Strategy 2. The green squares indicate high-value objects. The coverage preferences are 0.95 inside the objects and 0.8 outside.

appropriately, without extensive uncovered spots, as one can see in Figure 5 (b).

In addition to high-value areas, there may exist forbidden areas or obstacles, where no sensors are allowed. One can distinguish two types of obstacles. The first consists of obstacles that should be covered with certain preferences, although no sensors are allowed inside obstacles. The second consists of obstacles within which coverage is irrelevant. One can think about such obstacles as if they have the coverage preferences equal to zero. In this case, the main goal of coverage will be satisfying the preferences

outside the obstacles.[4] Figure 6 presents the sensor placement by Strategy 1 and Strategy 2 in the presence of high-value areas (green squares) and obstacles (black squares). Obstacles of the first type can be seen in Figure 6, (a) and (b). For this case, the coverage preferences were 0.95 inside the green squares and 0.8 everywhere else including the interior of the obstacles. The placement for the obstacles of the second type is shown in Figure 6, (c) and (d). For this case, the required coverage was 0.95 inside the high-value areas and, effectively, zero inside the obstacles.

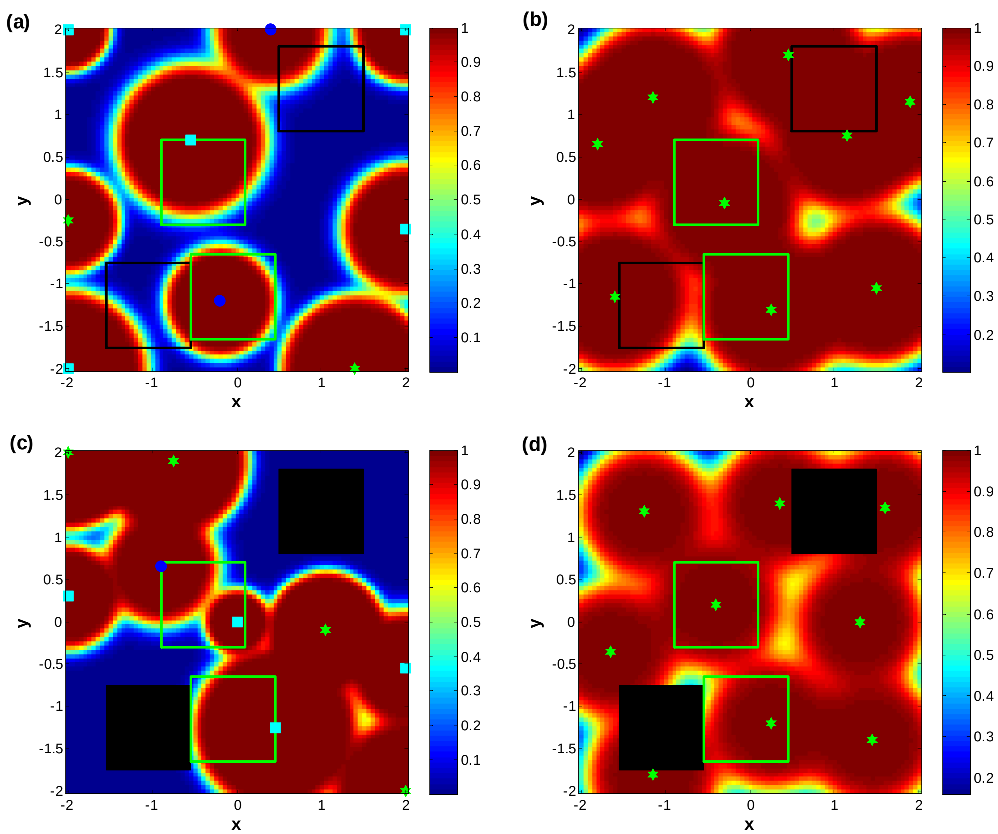


Figure 6. Sensor locations in the presence of high-value objects (green squares) and obstacles (black squares). (a) Strategy 1, obstacles of the first type. (b) Strategy 2, obstacles of the first type. (c) Strategy 1, obstacles of the second type. (d) Strategy 2, obstacles of the second type.

[4]In practice, such objects may represent natural nooks or objects with independent (internal) guarding.

The most complicated, yet still practically important, case is when the obstacles coincide with the areas of high importance. That is, one would like most to cover a certain area where no sensors are allowed. Strategy 1 cannot handle this case because this strategy places a sensor into the worst covered place, which, in this case, happens to be forbidden. Therefore, Strategy 1 will place sensors in other worst covered places, irrelevant to the areas with high importance. The only way to make Strategy 1 succeed in this case is to specify a high-value area wider than the forbidden one, so that at least the edge would be accessible, as shown in Figure 7 (a). In contrast, Strategy 2 succeeds without this additional restriction. It can handle the situations when the forbidden areas are wider, narrower, or exactly equal to the high-value areas. For example, in Figure 7 (b), the obstacle exactly coincides with the high-value area so that it is seen as one green square.

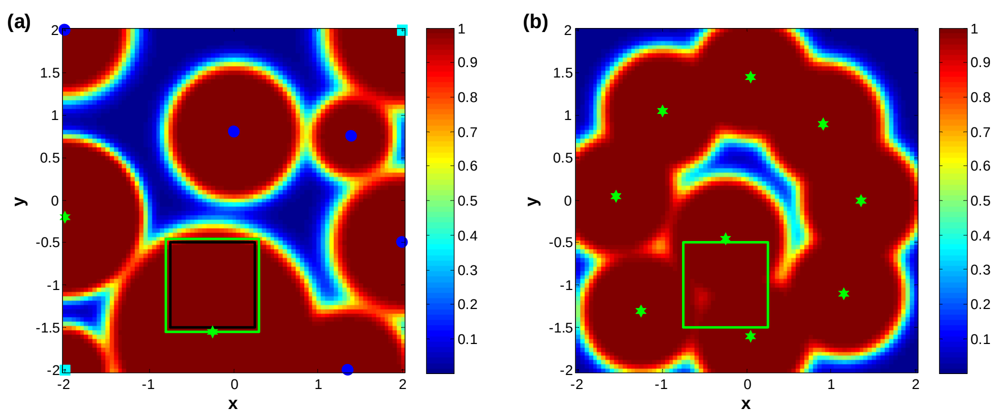


Figure 7. Sensor placement by (a) Strategy 1 and (b) Strategy 2 when the obstacle (black square) coincides with the high-value area (green square). Strategy 1 cannot handle the case unless the high-value area is wider than the obstacle.

6 Safe-Mode Concept

The safe-mode concept, presented in this section, is an application of the voting logic inference (Klein 2007) to the probabilistic sensor framework. Although a more natural application of this inference would imply a sensor performance measure other than the probability of detection (e.g., a degree of matching of a received signal to the ideal target signature or number of matching signatures for multisignature targets), it still can be applied to this one.

Sensor performance depends on operational conditions (e.g., combinations of terrain, weather, and atmospheric conditions) at a specific location. For example, performance of the acoustic sensor depends substantially on wind and time of day (Pettit and Wilson 2007). In other words, the radius of visibility is not static, it depends on environmental conditions. The most conservative way to account for this performance variability is to always use the sensor's worst anticipated characteristics in a placement strategy. However, operational conditions that are worst for one type of sensor may be best for another, so that a situation when all of the sensors have their worst performance simultaneously may never occur. For example, visible-spectrum cameras are unlikely to see a distant object at night while an acoustic sensor has the highest performance at this time. The safe-mode concept can help to overcome this issue.

Suppose that the required probability of detection of a sensor network is equal to or greater than 0.9 (at some fixed probability of false alarm). In the case of two independent sensors, there are three modes when this requirement is satisfied: either one or the other sensor has a probability of detection over 0.9, or both of them have a probability of detection in the range $0.7-0.9$. Each of these modes is safe because it matches the total probability of detection required. Thus, a safe mode can be specified by a requirement that the individual sensor's probability of detection belong to certain bands in the probability space. For example, Table 1 demonstrates these bands for three independent sensors, and the definition of the safe modes for this case is given in Table 2.

Table 1. Band definitions for three independent sensors.

Band		P_d	P_{fa}
1	Uncertain detection	0–0.55	10^{-6}
2	Low confidence	0.55–0.7	10^{-6}
3	Medium confidence	0.7–0.9	10^{-6}
4	High confidence	0.9–1	10^{-6}

Table 2. Definition of safe modes for the case shown in Table 1.

Safe Mode	Sensor		
	1	2	3
1	Band 2	Band 2	Band 2
2	Band 3	Band 3	
3		Band 3	Band 3
4	Band 3		Band 3
5	Band 4		
6		Band 4	
7			Band 4
Each of the safe modes provides a probability of detection greater than 0.9.			
Blank spaces signify any probability of detection value.			

As operational conditions vary, the sensors' probabilities of detection also vary. Suppose there are H combinations of weather/terrain conditions for which P_d of each sensor has been calculated. (In practice, this could be a tedious task). This H is assumed to be large enough to give a robust statistic about operational conditions. Having a distribution of detection probabilities P_d, one can calculate the probability that a sensor's P_d belongs to a certain band. Figure 8 shows an example of such calculations for three independent sensors. Using this information, the main question could be addressed: what is the probability that at least one of the safe modes occurs (with respect to possible operational conditions)?

To answer this question, one should calculate the probability of any of the safe modes that are not mutually independent or exclusive (the bands for the same sensor are exclusive because they are chosen with no overlapping, but the modes are not). The answer also depends on the location, meaning that the safe-mode probability is different for different spatial points. This makes it possible to use the safe-mode probability in a placement strategy instead of the probability of detection. The optimal

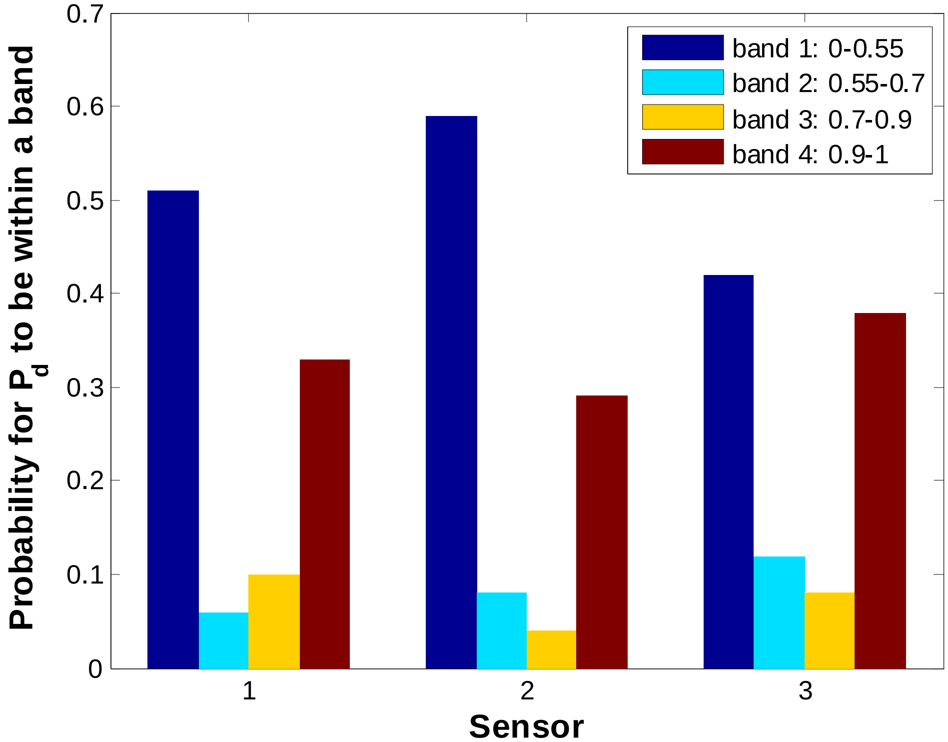


Figure 8. Probability that the probability of detection of a single sensor falls into a certain band. The statistics is obtained from 100 random numerical realizations of signal and noise parameters.

sensor locations, then, should maximize the safe-mode probability, which means a high robustness relative to arbitrary weather/terrain conditions. Figure 9 shows an example of how an average safe-mode probability depends on the number of sensors in a network. The red line indicates the desired level of the safe-mode probability (equal to 0.95 in this example). Note that if the safe-mode probability equals one, then, under any operational conditions, at least one safe mode will occur.

The application of this concept to a real network of sensors could be limited by computational time. The main problem is an exponential growth of safe modes as the number of sensors increases. Figure 10 demonstrates the assessment of the computational time (on an Intel Xeon 2.66 GHz PC with 4 Gb of RAM). The plot consists of the actual elapsed

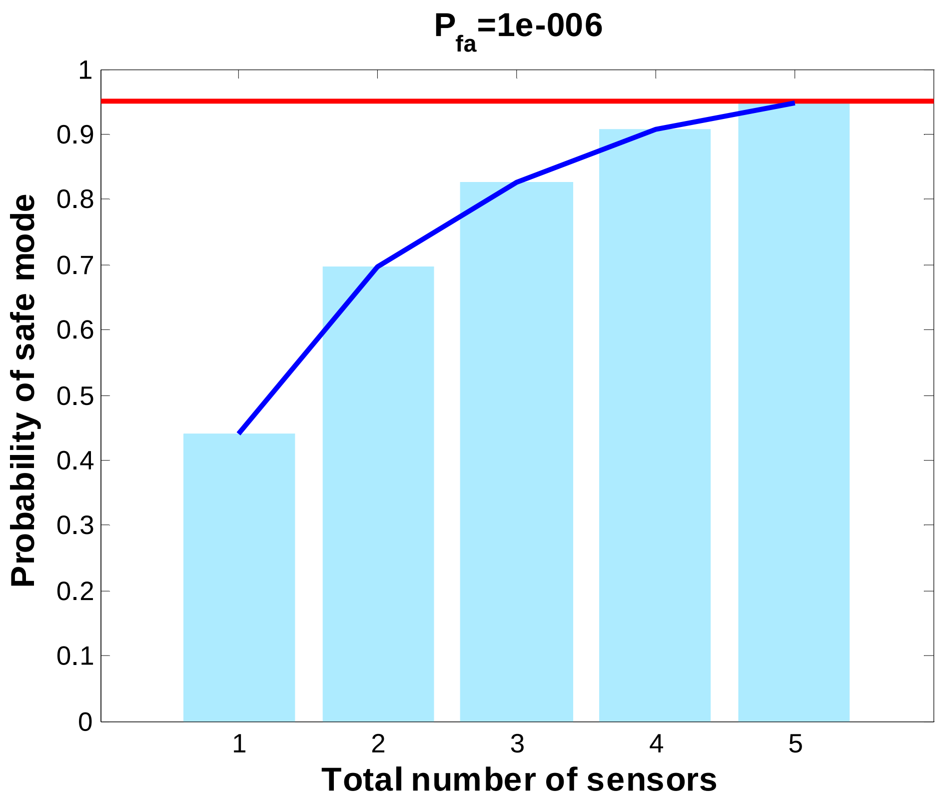


Figure 9. Average probability of safe mode versus number of sensors in a network.

time (the blue curve with markers) and the prediction (the red curve with no markers) derived from a curve fitting model (the first experimental point was not included in the model). The elapsed computational time t_e is proportional to $\exp[M^{1.5}]$, where M is the total number of sensors in a network. According to Figure 10, one would need approximately eight full days to process eight sensors. A code optimization may slightly reduce the time required, but the fact that the safe modes are not independent prevents effective calculations of the safe-mode probability. The practical applications of this concept directly are limited to seven to eight sensors. One way to incorporate this concept for a larger number of sensors is to place sensors in accordance with the probability of detection, as described in sections 3 and 4, and then model how this probability changes due to various operational conditions (already at fixed sensor locations). Note that this approach can assess the safe-mode probability for a given sensor's configuration but cannot derive the most robust locations.

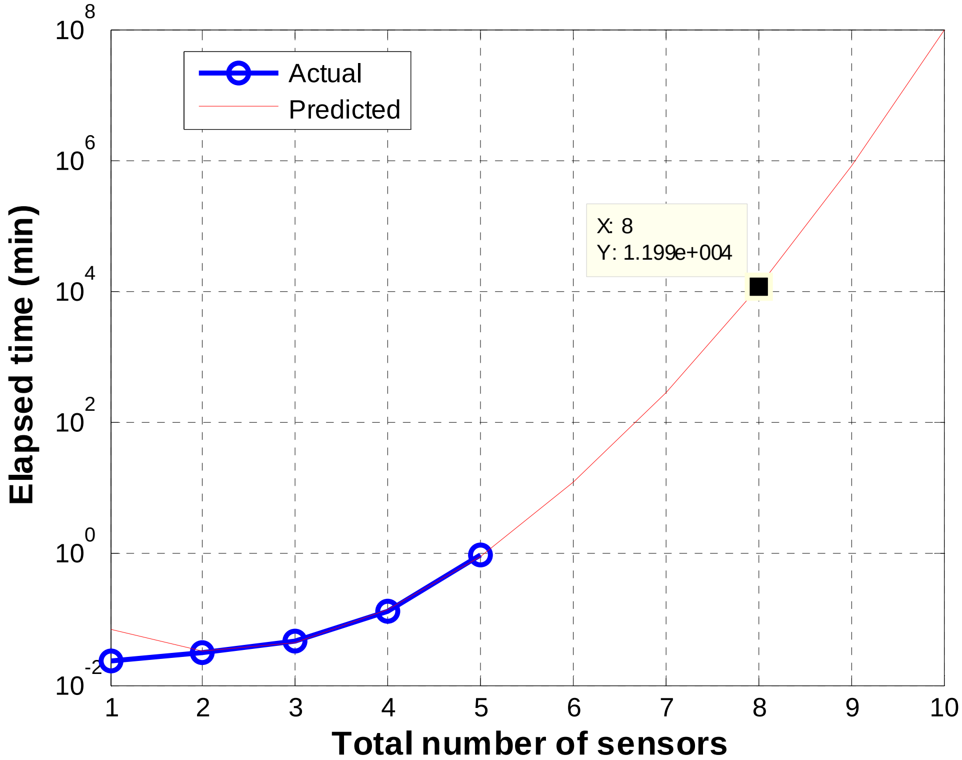


Figure 10. Actual (blue curve with markers) and predicted (red curve) computational times required for safe-mode solutions versus the total number of sensors.

7 Summary and Conclusions

This technical report presents several results regarding an optimal sensor placement. Sensor performance is evaluated in terms of the probability of detection (at some given probability of false alarm). A physical model of signal propagation from a source to a sensor is developed, allowing one to obtain a realistic spatial dependence of the probability of detection. It is unlikely that this dependence, shown in Figure 2, can be described by the exponential or Gaussian functions.

Two strategies for optimal sensor placement were presented. Strategy 1 is heuristic and fast, and Strategy 2 is a solution of the strict binary linear programming problem. Both strategies incorporate the coverage preferences that are formulated as the desired probability of detection and can be nonuniform (allowing one to assign the higher probability of detection for high-value objects). Strategy 1 places sensors one-by-one in the location where they are most needed (the worst covered and the farthest away from other sensors) while Strategy 2, strictly speaking, does not allow the sequential placement and yields the global optimum of total costs associated with sensors. These costs can include not only the number of sensors but also other disincentives, such as the actual price of sensors, their vulnerability, or their life-cycle costs. If the spatial resolution is fine, a strict solution of Strategy 2 becomes impractical because of the computational time required. For this case, a fast algorithm for an approximate solution was developed.

To compare these two strategies, a numerical experiment was conducted. In this experiment, several cases were studied, such as the coverage of a uniform area, an area with high-value objects, and an area with obstacles. The most complicated case, yet of great practical importance, was also studied, where the obstacles coincide with high-value objects. The numerical experiment revealed that the approximate solution of the strict optimization problem (Strategy 2) outperforms a strict solution of the approximate optimization problem (Strategy 1).

The safe-mode concept, described in this report, views the sensor placement problem from the standpoint of robustness relative to different op-

erational conditions. Operational conditions are defined by combinations of terrain, weather, and daytime conditions in the place where a sensor is located. Potentially, this concept allows one to find the most robust sensor placement. However, direct application of this concept depends significantly on the total number of sensors and is, in practice, limited to several sensors. One way to bypass this limitation is to use worst-case characteristics of each sensor in the placement strategies accounting for the probability of detection. This will provide a high value of the safe-mode probability, although may be excessive because the situation when all of the sensors have their worst characteristics may never occur. Another way to adopt the safe-mode concept to a greater number of sensors is to employ distributed calculations (under condition that an algorithm for safe modes can be parallelized). This possibility was not studied in this technical report.

References

Balanis, C. A. 1989. *Advanced engineering electromagnetics*. New York: J. Wiley & Sons.

Burdic, W. S. 1984. *Underwater acoustic system analysis*. Englewood Cliffs, NJ: Prentice-Hall.

Dhillon, S. S., and K. Chakrabarty. 2003. Sensor placement for effective coverage and surveillance in distributed sensor networks. *Proc IEEE Wireless Communications and Networking Conference* 3:1609–1614.

Eidenbenz, S. 2002. Approximation algorithms for terrain guarding. *Inf Process Lett* 82:99–105.

Elnagar, A., and L. Lulu. 2005. An art gallery-based approach to autonomous robot motion planning in global environments. *Proc IEEE/RSJ International Conference on Intelligent Robots and Systems* 2079–2084.

Erdem, U. M., and S. Sclaroff. 2004. Optimal placement of cameras in floor plans to satisfy task requirements and cost constraints. In *Proceedings, 5th Workshop on Omnidirectional Vision, Camera Networks and Non-Classical Cameras, May 16, 2004*. Prague, Czech Republic.

González-Banos, H., A. Efrat, J. C. Latombe, E. Mao, and T. M. Murali. 1999. Planning robot motion strategies for efficient model construction. In *Proceedings, 9th International Symposium on Robotics Research*. Salt Lake City, UT.

Ishimaru, A. 1978. *Wave propagation and scattering in random media*. New York: Academic Press. (Reissued 1997, New York: IEEE/OUP).

Klein, L. A. 2007. *Sensor and data fusion: a tool for information assessment and decision making*. Washington, DC: SPIE Press.

McKenna, J. 1987. Worst-case optimal hidden-surface removal. *ACM Trans Graphics* 6:19–28.

Mittal, A., and L. S. Davis. 2008. A general method for sensor planning in multi-sensor systems: extension to random occlusion. *Int J Computer Vision* 76(1):31–52.

O'Rourke, J. 1987. *Art gallery theorems and algorithms*. New York: Oxford University Press.

Ostashev, V. E. 1997. *Acoustics in moving inhomogeneous media*. London, UK: E&FN Spon.

Pettit, C. L., and D. K. Wilson. 2007. Proper orthogonal decomposition and cluster weighted modeling for sensitivity analysis of sound propagation in the atmospheric surface layer. *J Acoust Soc Am* 122(3):1374–1390.

Shermer, T. C. 1992. Recent results in art galleries. *Proc IEEE* 80(9):1384–1399.

Sierksma, G. 2002. *Linear and integer programming: theory and practice*, 2nd Ed. New York; Basel, Switzerland: Marcel Dekker.

Tatarskii, V. I. 1971. *The effects of the turbulent atmosphere on wave propagation*. Jerusalem, Israel: Keter Publishing House.

Wilson, D. K., C. L. Pettit, M. S. Lewis, S. Mackay, and P. Seman. 2008. Probabilistic framework for characterizing uncertainty in the performance of networked battlefield sensors. *Proc SPIE* 698104:1–12.

Wixson, L. 1994. Viewpoint selection for visual search. *Proc IEEE/Conference on Computer Vision and Pattern Recognition* 800–805.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)* December 2008	**2. REPORT TYPE** Technical Report	**3. DATES COVERED** *(From - To)*
4. TITLE AND SUBTITLE Optimal Sensor Placement with Terrain-Based Constraints and Signal Propagation Effects		**5a. CONTRACT NUMBER** **5b. GRANT NUMBER** **5c. PROGRAM ELEMENT NUMBER**
6. AUTHOR(S) Sergey N. Vecherin, D. Keith Wilson, and Chris L. Pettit		**5d. PROJECT NUMBER** **5e. TASK NUMBER** **5f. WORK UNIT NUMBER**
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) U.S. Army Engineer Research and Development Center Cold Regions Research and Engineering Laboratory 72 Lyme Road Hanover, NH 03755-1290		**8. PERFORMING ORGANIZATION REPORT NUMBER** ERDC/CRREL TR-08-24
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)		**10. SPONSOR/MONITOR'S ACRONYM(S)** **11. SPONSOR/MONITOR'S REPORT NUMBER(S)**

12. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

The optimal sensor placement problem, as considered here, is to select the types and locations of sensors providing coverage at high-value terrain locations while minimizing a specified cost function. The cost function can reflect various disincentives, such as the actual cost of the sensors, the total number of sensors, and the probability that the sensor will be found and disabled by hostile actors. The probability of detection (at a certain probability of false alarm) is assumed to depend on terrain conditions and obstructions, and may be arbitrarily complex. Two strategies are described for finding the minimal number of sensors, and their locations that will satisfy given coverage preferences. The first is heuristic in nature and based on placing sensors one-by-one where the probability of detection is minimal. This strategy offers a rapid, but suboptimal solution. The second strategy is based on solution of the binary linear programming problem. For the case of fine spatial resolution that leads to large matrix dimensions, a fast algorithm for approximate solution of this problem is developed. The key features of this study are: 1) the probabilistic framework of sensor performance, 2) incorporation of the coverage preferences in the placement strategy, 3) realistic modeling and incorporation of the sensors' probability of detection, 4) multimodal sensor support, 5) a strict formulation of the optimal coverage problem, 6) development of a fast algorithm for approximate solution of the binary linear programming problem, and 7) introduction of a safe-mode concept.

15. SUBJECT TERMS
binary linear programming, multimodal sensor network, optimal sensor placement, probabilistic framework, probability of detection, safe-mode concept, sensor location optimization, sensor performance, signature propagation effects, weather effects on sensors

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			**19b. TELEPHONE NUMBER** *(include area code)*
U	U	U	U	45	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. 239.18